

Answer the following true/false questions by circling the correct word.

1. A common denominator is needed to multiply rational expressions. True False
2. A common denominator is needed to add or subtract rational expressions. True False
3. The same rules apply to fractions with variables and fractions without variables. True False

9.4

Multiplication of Rational Expressions

- 1) Factor the numerators and denominators so there is only multiplication
- 2) Cancel anything in the numerator with anything in the denominator

Division of Rational Expressions

- 1) Flip any fraction being divided and change to a multiplication problem

Perform the indicated operation. Simplify the result.

$$4. \frac{x^2}{2} \cdot \frac{2}{x} = \left(\frac{x}{2} \right)$$

$$5. \frac{(x+1)}{x^2} \cdot \frac{x^2}{(x+1)(x+1)} = \left(\frac{x^2}{(x+1)} \right)$$

$$6. \frac{x+5}{x} \div \frac{x+5}{2x} = \left(2 \right)$$

$$7. \frac{2(x+4)}{-3x} \cdot \frac{3x}{8(x+4)} = \left(-\frac{3}{4x} \right)$$

$$8. \frac{4x^2y^3}{x^5y^2} \cdot \frac{520}{520} = \left(\frac{1}{5x^3y^2} \right)$$

$$9. \frac{9x^4}{y^4} \cdot \frac{x^2}{436y} = \left(\frac{9x^6}{4y^5} \right)$$

$$10. \frac{12x^2y}{5y^2} \div \frac{3x^2}{2xy}$$

$$\frac{4 \cancel{12} x^2 \cancel{y}}{5 \cancel{y}^2} \cdot \frac{2 \cancel{xy}}{3 \cancel{x}^2}$$

$$\left(\frac{8x}{5} \right)$$

$$11. \frac{x^2-3x+2}{25x} \div \frac{x-1}{5x^2}$$

$$\frac{(x-2)(x-1)}{5 \cancel{25}} \cdot \frac{5x^2}{(x-1)}$$

$$\left(\frac{x(x-2)}{5} \right)$$

$$12. \frac{5x^2-20}{25x^2} \div \frac{x^2+6x+8}{x^2+10x+24}$$

$$5(x^2-4)$$

$$\frac{5(x-2)(x+2)}{5 \cancel{25} x^2} \cdot \frac{(x+6)(x+4)}{(x+4)(x+2)}$$

$$\left(\frac{(x-2)(x+6)}{5x^2} \right)$$

Adding/Subtracting Rational Expressions

- 1) Factor the denominators
 - 2) Find the LCD
 - 3) Multiply by $\frac{\text{missing}}{\text{missing}}$
 - 4) Multiply out the numerators
 - 5) Combine Like Terms
- Keep the denominators the same

$$13. \frac{5x^2 - 8x}{x^2 - 9} - \frac{4x + 9x^2}{x^2 - 9}$$

$$\frac{4x^2 - 12x}{x^2 - 9} = \frac{-4x(x+3)}{(x+3)(x-3)} = \frac{-4x}{x-3}$$

$$14. \frac{3x+1}{x+3} + \frac{2x-1}{x+3}$$

$$\frac{5x}{x+3}$$

$$15. \frac{7}{x-2} - \frac{x}{x-2}$$

$$\frac{7-x}{x-2}$$

$$16. \frac{x}{x^2 + x - 2} + \frac{1}{x+2} \cdot \frac{(x-1)}{(x-1)}$$

$$(x+2)(x-1)$$

$$\text{LCD: } (x+2)(x-1)$$

$$\frac{x}{(x+2)(x-1)} + \frac{x-1}{(x+2)(x-1)}$$

$$\frac{2x-1}{(x+2)(x-1)}$$

$$17. \frac{x}{x^2 - x - 30} - \frac{1}{x+5} \cdot \frac{(x-6)}{(x-6)}$$

$$(x-6)(x+5)$$

$$\text{LCD: } (x-6)(x+5)$$

$$\frac{x}{(x-6)(x+5)} - \frac{x-6}{(x+5)(x-6)}$$

$$\frac{6}{(x-6)(x+5)}$$

$$18. \frac{5x}{x^2 - 2x - 15} + \frac{2x}{x-5} \cdot \frac{(x+3)}{(x+3)}$$

$$(x-5)(x+3)$$

$$\text{LCD: } (x-5)(x+3)$$

$$\frac{5x}{(x-5)(x+3)} + \frac{2x^2 + 6x}{(x-5)(x+3)}$$

$$\frac{2x^2 + 11x}{(x-5)(x+3)}$$

$$19. \frac{(x+3)}{(x+3)} \cdot \frac{2x}{(x-6)} - \frac{3}{(x+3)} \cdot \frac{(x-6)}{(x-6)}$$

$$\text{LCD: } (x-6)(x+3)$$

$$\frac{2x^2 + 6x}{(x-6)(x+3)} - \frac{3x-18}{(x-6)(x+3)}$$

$$\frac{2x^2 + 3x + 18}{(x-6)(x+3)}$$

Simplify the complex fraction.

$$20. \frac{5 + \frac{1}{4}}{2 + \frac{2}{3}} \quad \text{LCD: 4} \quad \frac{4 \cdot \frac{5}{4} + \frac{1}{4}}{\frac{20}{4} + \frac{1}{4}} = \frac{5 + \frac{1}{4}}{2 + \frac{2}{3}} \quad \text{LCD: 3} \quad \frac{\frac{6}{3} + \frac{2}{3}}{\frac{21}{4} + \frac{1}{4}} = \frac{\frac{8}{3}}{\frac{21}{4}} = \frac{21}{4} \cdot \frac{3}{8} = \frac{63}{32}$$

$$22. \frac{\frac{x+3}{3x^2}}{\frac{6x^2}{(x+3)^2}} = \frac{(x+3)}{3x^2} \div \frac{6x^2}{(x+3)^2} = \frac{(x+3)}{3x^2} \cdot \frac{(x+3)(x+3)}{6x^2} = \frac{(x+3)^3}{18x^4}$$

$$24. \frac{\frac{x}{8} - \frac{3}{4x}}{\frac{4x}{x+4} - \frac{5}{x}} \quad \text{LCD: } 8x \quad \frac{\frac{x^2}{8} - \frac{6}{8x}}{\frac{4x^2}{x(x+4)} - \frac{5x+20}{x(x+4)}} = \frac{\frac{x^2-6}{8x}}{\frac{4x-5x-20}{x(x+4)}} = \frac{x^2-6}{8x} \div \frac{-x-20}{x(x+4)} = \frac{x^2-6}{8x} \cdot \frac{x(x+4)}{-x-20} = \frac{(x^2-6)(x+4)}{-8x-160}$$

$$21. \frac{\frac{x-4}{3}}{5 + \frac{1}{x}}$$

$$\frac{\frac{x}{3} - \frac{4}{3}}{5 + \frac{1}{x}} \quad \text{LCD: 3} \quad \frac{\frac{x}{3} - \frac{12}{3}}{\frac{5x}{x} + \frac{1}{x}} = \frac{\frac{x-12}{3}}{\frac{5x+1}{x}} = \frac{x-12}{3} \div \frac{5x+1}{x} = \frac{x-12}{3} \cdot \frac{x}{5x+1} = \frac{x(x-12)}{3(5x+1)}$$

$$23. \frac{\frac{2}{x^2-4} - \frac{1}{x+2}}{\frac{2}{4x-12}}$$

$$\frac{\frac{2}{x^2-4} - \frac{1}{x+2}}{\frac{2}{4x-12}} \quad \text{LCD: } (x-2)(x+2) \quad \frac{\frac{2}{(x+2)(x-2)} - \frac{1}{x+2}}{\frac{2}{4(x-3)}} = \frac{\frac{2}{(x+2)(x-2)} - \frac{x-2}{(x+2)(x-2)}}{\frac{2}{4(x-3)}} = \frac{\frac{-x+4}{(x+2)(x-2)}}{\frac{2}{4(x-3)}} = \frac{(-x+4)}{(x+2)(x-2)} \cdot \frac{2(x-3)}{2} = \frac{2(-x+4)(x-3)}{(x+2)(x-2)}$$

$$25. \frac{\frac{1}{6} + \frac{1}{8x}}{\frac{4x}{x-2} + \frac{6}{2x-4}}$$

$$\frac{\frac{1}{6} + \frac{1}{8x}}{\frac{4x}{x-2} + \frac{6}{2x-4}} \quad \text{LCD: } 24x \quad \frac{\frac{4x}{4x} \cdot \frac{1}{6} + \frac{1}{8x} \cdot \frac{3}{3}}{\frac{2 \cdot 4x}{2(x-2)} + \frac{6 \cdot 6}{2(x-2)}} = \frac{\frac{4x}{24x} + \frac{3}{24x}}{\frac{8x}{2(x-2)} + \frac{36}{2(x-2)}} = \frac{\frac{4x+3}{24x}}{\frac{8x+36}{2(x-2)}} = \frac{4x+3}{24x} \div \frac{8x+36}{2(x-2)} = \frac{4x+3}{24x} \cdot \frac{2(x-2)}{8x+36} = \frac{(4x+3)(x-2)}{24x(4x+9)}$$

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Solving Rational Expressions

- 1) Find the LCD
(factor the denominator if necessary)
- 2) Multiply each term by the LCD
- 3) Cancel what you can
- 4) Solve for x
 - a. If x^2 problem: set = 0
 - b. If x problem: get x 's on one side and the numbers on the other side

Solve the equation using any method. Check each solution.

$$26. \frac{3x}{4} = \frac{(x+1)}{2}$$

$$(4)(x+1) = (3x)(2)$$

$$4x+4 = 6x$$

$$4 = 2x$$

$$2 = x$$

$$28. \frac{2x-9}{(x-7)} + \frac{x}{2} = \frac{5}{(x-7)}$$

$$LCD = 2(x-7)$$

$$2(2x-9) + x(x-7) = 10$$

$$4x-18 + x^2 - 7x = 10$$

$$x^2 - 3x - 18 = 10$$

$$x^2 - 3x - 28 = 0$$

$$(x-7)(x+4) = 0$$

$$x = 7, -4$$

$$27. \frac{10}{(x+3)} + \frac{10}{8} = \frac{6}{1}$$

$$LCD: 3(x+3)$$

$$30 + 10(x+3) = 18(x+3)$$

$$30 + 10x + 30 = 18x + 54$$

$$60 + 10x = 18x + 54$$

$$6 = 8x$$

$$\frac{6}{8} = x$$

$$x = \frac{3}{4}$$

$$29. \frac{2}{x} = \frac{x}{x^2-8}$$

$$x^2 = 2(x^2-8)$$

$$x^2 = 2x^2 - 16$$

$$-x^2 = -16$$

$$x^2 = 16$$

$$x = \pm 4$$

$$30. \frac{4 \cdot \cancel{(x+4)} \cdot \cancel{(x-1)}}{\cancel{x^2+3x-4}} + 3 = \frac{7}{\cancel{(x+4)}} \cdot \cancel{(x+4)} \cdot \cancel{(x-1)}$$

$$(x+4)(x-1)$$

$$\text{LCD: } (x+4)(x-1)$$

$$4 + 3(x^2 + 3x - 4) = 7(x-1)$$

$$4 + 3x^2 + 9x - 12 = 7x - 7$$

$$3x^2 + 9x - 8 = 7x - 7$$

$$3x^2 - 2x - 1 = 0$$

$$(3x+1)(x-1) = 0$$

$$x = -\frac{1}{3}, 1$$

$$31. \frac{(x+3) \cdot \cancel{(x-4)} \cdot \cancel{(x+6)}}{\cancel{(x-4)} \cdot \cancel{(x+6)}} = \frac{(5x+14)}{\cancel{x^2+2x-24}}$$

$$(x-4)(x+6)$$

$$\text{LCD: } (x-4)(x+6)$$

$$(x+3)(x+6) = 5x+14$$

$$x^2 + 9x + 18 = 5x + 14$$

$$x^2 + 4x + 4 = 0$$

$$(x+2)(x+2) = 0$$

$$x = -2$$

APPLICATIONS

32. At a factory, gallons of paint are produced by a machine in accordance to the formula $G = 40h \cdot \frac{560}{h}$ where G = the number of gallons produced, and h = the number of hours the paint-making machine works. How many hours does the machine need to work to produce 200 gallons of paint?

$$200 = 40h \cdot \frac{560}{h}$$

$$\text{LCD: } h$$

$$200h = 40h^2 \cdot 560$$

$$0 = \frac{40h^2 - 200h \cdot 560}{40}$$

$$0 = h^2 - 5h - 14$$

$$0 = (h-7)(h+2)$$

$$h = 7, -2 \text{ hours}$$

can't have
-2 hours

33. The number of bottles produced by a machine at a plant is given by the equation $b = \frac{20m^3 - 120m^2}{m^2 - m - 30}$.

B = the number of bottles, and m = the number of minutes the machine works.

a) Simplify the expression above

$$b = \frac{20m^2 \cdot \cancel{(m-6)}}{\cancel{(m-6)}(m+5)} = \frac{20m^2}{m+5}$$

b) If the plant's machine works for 10 minutes, how many bottles does it produce?

$$10 = m$$

$$b = \frac{20m^2}{m+5}$$

$$b = \frac{20(10)^2}{10+5} = \frac{20(100)}{15} = 133.33 \text{ bottles}$$